

If $\vec{r} = 3\vec{j} - 2\vec{k}$ and $\vec{s} = 5\vec{i} - 4\vec{j} + c\vec{k}$ are orthogonal, find the value of c .

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$$\vec{r} \cdot \vec{s} = 0$$

$$0(5) + 3(-4) - 2(c) = 0$$

$$-12 - 2c = 0$$

$$c = -6$$

MULTIPLE CHOICE: Circle the correct answer.

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If $\|\vec{x}\| = 7$, $\|\vec{y}\| = 6$ and the angle between \vec{x} and \vec{y} is $\frac{5\pi}{6}$, then $\vec{x} \cdot \vec{y} =$

$$\|\vec{x}\| \|\vec{y}\| \cos \frac{5\pi}{6}$$

[a] -21

[b] $21\sqrt{2}$

[c] $-21\sqrt{3}$

[d] 42

[e] none of the above

Let P be the point $(8, -5)$, and Q be the point $(-1, -2)$.

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Let \vec{m} be the vector with terminal point P and initial point Q .

[a] Find a unit vector perpendicular to \vec{m} . Write your final answer in component form.

$$\vec{m} = \langle 9, -3 \rangle$$

$$\langle 9, -3 \rangle \cdot \langle a, b \rangle = 0$$

$$9a - 3b = 0$$

$$a = 1, b = 3$$

$$\frac{1}{\|\langle 1, 3 \rangle\|} \langle 1, 3 \rangle = \frac{1}{\sqrt{10}} \langle 1, 3 \rangle \\ = \left\langle \frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right\rangle$$

[b] A force represented by the vector $\langle 2, -4 \rangle$ moves an object from Q to P . Find the work done.

$$\langle 2, -4 \rangle \cdot \langle 9, -3 \rangle = 18 + 12 = 30$$

Let $\vec{p} = -3\vec{j} + \vec{k}$ and $\vec{q} = -\vec{i} - 2\vec{j} + 2\vec{k}$.

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[a] Find a vector of magnitude 5 in the opposite direction as \vec{q} .

$$-\frac{5}{\|\vec{q}\|} \vec{q} = \frac{-5}{3} \langle -1, -2, 2 \rangle = \left\langle \frac{5}{3}, \frac{10}{3}, -\frac{10}{3} \right\rangle$$

[b] Write \vec{p} as the sum of two orthogonal vectors, one of which is $\text{proj}_{\vec{q}} \vec{p}$.

$$\begin{aligned} \text{proj}_{\vec{q}} \vec{p} &= \frac{\vec{p} \cdot \vec{q}}{\vec{q} \cdot \vec{q}} \vec{q} \\ &= \frac{0 + 6 + 2}{9} \langle -1, -2, 2 \rangle \\ &= \frac{8}{9} \langle -1, -2, 2 \rangle \\ &= \left\langle -\frac{8}{9}, -\frac{16}{9}, \frac{16}{9} \right\rangle \end{aligned}$$

$$\begin{aligned} &\langle 0, -3, 1 \rangle - \left\langle -\frac{8}{9}, -\frac{16}{9}, \frac{16}{9} \right\rangle \\ &= \left\langle \frac{8}{9}, -\frac{11}{9}, -\frac{7}{9} \right\rangle \\ \vec{p} &= \left\langle -\frac{8}{9}, -\frac{16}{9}, \frac{16}{9} \right\rangle \\ &\quad + \left\langle \frac{8}{9}, -\frac{11}{9}, -\frac{7}{9} \right\rangle \end{aligned}$$

[c] Find a vector perpendicular to both \vec{p} and \vec{q} .

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -3 & 1 \\ -1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} \\ 0 & -3 \\ -1 & -2 \end{vmatrix} = -6\vec{i} - \vec{j} + 2\vec{k} = \langle -4, -1, -3 \rangle$$

If $\vec{g} = a\vec{i} - 9\vec{j} + 4\vec{k}$ and $\vec{h} = 5\vec{i} - 6\vec{j} + b\vec{k}$ are parallel, find the values of a and b .

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$$\begin{aligned} \vec{g} &= c\vec{h} \\ \langle a, -9, 4 \rangle &= c \langle 5, -6, b \rangle \\ a &= 5c \longrightarrow a = 5\left(\frac{3}{2}\right) = \frac{15}{2} \\ -9 &= -6c \longrightarrow c = \frac{3}{2} \\ 4 &= bc \longrightarrow 4 = b\left(\frac{3}{2}\right) \longrightarrow b = \frac{8}{3} \end{aligned}$$

MULTIPLE CHOICE: Circle the correct answer.

SCORE: ____ / 9 PTS

If $\vec{b} \cdot \vec{c} = -76$, then the angle between \vec{b} and \vec{c} could be

ANGLE MUST BE OBTUSE

- [a] 0° [b] 14° [c] 90° [d] 166° [e] 284°

Let D be the point $(3, -1, -2)$, E be the point $(1, 3, -1)$ and F be the point $(2, -2, 1)$.

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- [a] Find the area of the triangle DEF .

$$\begin{aligned}\vec{DE} &= \langle -2, 4, 1 \rangle \\ \vec{DF} &= \langle -1, -1, 3 \rangle \\ \vec{DE} \times \vec{DF} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 4 & 1 \\ -1 & -1 & 3 \end{vmatrix} = 12\vec{i} - \vec{j} + 2\vec{k} \\ &\quad + \vec{i} + 6\vec{j} + 4\vec{k} \\ &= \langle 13, 5, 6 \rangle\end{aligned}$$

$$\frac{1}{2} \|\langle 13, 5, 6 \rangle\| = \frac{1}{2} \sqrt{230}$$

- [b] Find the general form of the equation of the plane containing D , E and F .

$$13(x-3) + 5(y+1) + 6(z+2) = 0$$

$$13x + 5y + 6z = 22$$

- [c] Find the symmetric equation of the line through E and parallel to the line with parametric equations $y = 21t + 11$.

$$x = 13t - 15$$

$$z = 17 - 19t$$

$$\frac{x-1}{13} = \frac{y-3}{21} = \frac{z+1}{-19}$$

DIRECTION VECTOR
 $\langle 13, 21, -19 \rangle$

- [d] Find parametric equations of the line through F and perpendicular to the plane $7x - 9z = 4$.

$$x = 2 + 7t$$

$$y = -2$$

$$z = 1 - 9t$$

NORMAL VECTOR
 $\langle 7, 0, -9 \rangle$

Vectors \vec{d} and \vec{e} are shown below. Sketch $2\vec{e} - 3\vec{d}$.

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